

# Clusters of interacting receptors can stabilize synaptic efficacies

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**Synaptic weights store memories that can last a lifetime. Yet, memory depends on synaptic protein receptors that are recycled in and out of the membrane at a fast rate, possibly several times an hour. Several proposals to bridge this vast gap in time scales between memory and its molecular substrate have relied on bistable molecular switches. Here, we propose an alternative to this approach based on clusters of interacting receptors in the synaptic membrane. We show that such clusters can be metastable and that the lifetime of such clusters can be many orders of magnitude larger than the lifetime of the receptors of which they are composed. We also demonstrate how bidirectional synaptic plasticity can be implemented in this framework.**

memory | model | synaptic plasticity

Synaptic efficacies depend on the number and conformational states of receptor proteins. However, receptors have a limited dwell time in the synaptic membrane, and they recycle in and out of the synapse possibly several times an hour (1, 2). Conformational changes due to phosphorylation are also short lived and can be reversed by phosphatases and receptor turnover. Yet, synaptic strengths are the basis of learning and memory processes that can persist a lifetime. The central difficulty in understanding the stability of memory and learning arises because synaptic efficacies must be uniquely regulated at the level of individual synapses. This observation rules out many possible mechanisms that are solely based on whole-cell processes, such as the regulation of gene expression.

The fundamental problem of preserving synapse-specific synaptic efficacies for long times, orders of magnitude larger than the lifetime of their molecular substrates, was pointed out by Francis Crick (3), who proposed a molecular switch as a likely solution. This idea was extensively expanded and investigated by John Lisman *et al.* (4, 5). Lisman hypothesized that this problem can be solved by a molecular switch in the signal transduction pathway that regulates synaptic efficacy and proposed a specific mechanism based on autophosphorylation of calmodulin-dependent PK II (CaMKII) holoenzymes. Modeling studies show that autophosphorylation of CaMKII results in a positive feedback loop that can keep the enzyme in an active state despite dephosphorylation by phosphatases and protein turnover (4, 5). The CaMKII hypothesis is appealing because it is well established that CaMKII and its autophosphorylation plays a key role in the induction of long-term potentiation (LTP) (6). However, there is no significant experimental evidence demonstrating that activation or autophosphorylation of CaMKII is necessary for the long-term maintenance of synaptic efficacies (7). Other components of the molecular signal transduction pathways controlling synaptic plasticity have also been proposed as possible molecular switches (8).

In this paper, I propose a theory in which the stability of synaptic efficacies is based on local interactions between receptors within a single synapse. Specifically, I propose that interactions between receptors within a cluster can alter the trafficking of receptors in and out of the synaptic membrane, thereby creating a metastable synaptic state that significantly increases the stability of synaptic efficacy without changing the mean dwell time of receptors in the synaptic membrane. The cluster theory proposed here is formally

distinct from equilibrium theories of synaptic stability because it does not result in equilibrium states that are stable forever. The synaptic states generated by the cluster theory are metastable; at some point in time, these states will break and decay. However, the lifetimes of these metastable states are orders of magnitude larger than the lifetimes of their components.

This paper also demonstrates how bidirectional and synapse-specific long-term plasticity can be incorporated into the model. Finally, I show that statistical fluctuations in the number of receptors are a signature of this model that might be used to distinguish it from other synaptic models.

The cluster theory of synaptic stability is presented here in an abstract form. However, if the general principals of this theoretical model are found to be consistent with experimental evidence, identifying the molecular basis of the cluster model will become important.

## Mathematical Methods

The variable  $S_{ij}$  is an occupation variable of the lattice site denoted by indices  $i$  and  $j$ . If the site is occupied,  $S_{ij} = 1$ ; otherwise,  $S_{ij} = 0$ . Insertion of a new receptor into the membrane can occur at any unoccupied site in the lattice, and internalization of a receptor can occur only at occupied sites. In this formulation, internalization occurs at a fixed rate, independent of interaction with other receptors. I used a fixed internalization rate  $\mu = 1/\tau_{in}$  per unit time, which implies that the probability of internalizing a receptor at site  $(i, j)$  in a small time step  $\Delta t$  is

$$P^{in}(i, j, t:t + \Delta t) = S_{ij}\mu\Delta t. \quad [1]$$

Throughout this paper, we use  $\mu = 1$ , which implies that the mean dwell time of a receptor in the membrane is 1 unit of time. Typically, we use a time step  $\Delta t < 0.01$ , which is significantly smaller than the other time constants in this system.

Inserting a new receptor into an unoccupied site depends on the occupation in the vicinity of the unoccupied site. I calculate a "field"  $h_k(i, j)$  at each unoccupied site  $i, j$  that measures the number of membrane-embedded receptors in the local neighborhood. The equation defining the field ( $h_1(i, j)$ ) is described in Eq. 4.

The field  $h_k$  will determine the conditional probability of inserting a new receptor into an unoccupied site. A typical parameter used in our simulations is  $L_1 = 1.5$ ; however, similar stability is obtained for the range of  $L_1 = 1.2$ – $2.0$ . The lattice repulsion constant used for the second population is  $L_2 = 0.9$ .

To determine insertion probability we use

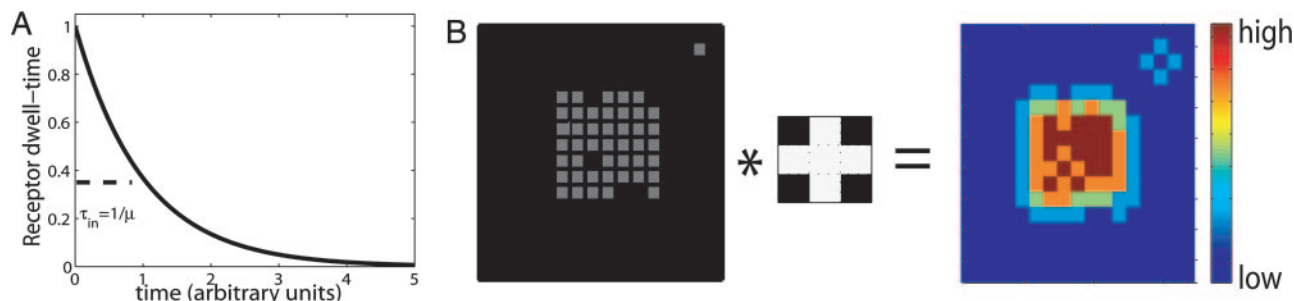
$$P_k(i, j) = 1/(1 + \exp(-\beta h_k(i, j))), \quad [2]$$

which varies smoothly from 0 to 1 as a function of  $h_k(i, j)$ . The constant  $\beta$  is the slope of this function. Stability increases for larger  $\beta$ . I typically use  $\beta = 50$ . However, values of  $\beta > 25$  are sufficient for stability of up to  $\approx 1,000$  time steps, with 49 receptors in the

Abbreviations: LTP, long-term potentiation; LTD, long-term depression; CaMKII, calmodulin-dependent PK II; AMPA,  $\alpha$ -amino-3-hydroxy-5-methyl-4-isoxazolepropionic acid.

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**Fig. 1.** Model assumptions. (A) Receptors in the synapse are internalized stochastically at a constant rate, and their probability of staying in the synapse decays exponentially with a time constant of 1. (B) The rate of insertion depends of the number of nearest neighbors. Given the occupation state (Left), a field is calculated (Right). The probability of inserting a new receptor is proportional to this field. The field can be computed from convolving the nearest-neighbor function (Center) with the state. The field is higher within the cluster and close to its boundaries than outside the cluster or near the isolated receptor.

initial state. This stability depends on other parameters, such as  $L_1$ . The probability of inserting a receptor in an unoccupied site in a very small time step  $\Delta t$  is then

$$P_k^{\text{ex}}(i, j) = (1 - S_{ij})(\rho_k r \Delta t P_k(i, j)), \quad [3]$$

where  $\rho_k$  is the probability that a receptor of type  $k$  is present in a position near the empty site, and  $r$  is the rate of transition into the empty site. Typically, we use  $\rho_1 = 0.95$  and  $r = 10$ , which implies that for  $P_k \approx 1$ , the average time for inserting a receptor into a vacant site with a high  $h_k$  is  $\approx 0.1$  units of time, significantly faster than the internalization rate and slower than the typical time step used. Eq. 3 is arrived at for small  $\Delta t$  by approximating the expression for finite  $\Delta t$ :  $\rho_k r \Delta t P_k(i, j) \approx 1 - \exp(-\rho_k r \Delta t P_k(i, j))$ .

The key to stability is not the identity of specific parameters, such as  $\rho_k$  and  $r$ , but their consequence that the characteristic time for insertion into an empty site in a cluster is much shorter than the characteristic time of removing a receptor from a cluster.

To reduce run time, we use parallel dynamics. The use of parallel dynamics is not a problem because we use small time steps in which a very small number of events occur across the whole lattice. I ran a few random sequential simulations and obtained indistinguishable results.

## Results

The cluster theory of synaptic stability is based on several assumptions: (i) Synaptic efficacy is proportional to the number of postsynaptic receptors [for example,  $\alpha$ -amino-3-hydroxy-5-methyl-4-isoxazolepropionic acid (AMPA) receptors]. (ii) Receptors in the postsynaptic density are clustered. (iii) The insertion rate of a receptor in the vicinity of other receptors in the cluster is much higher than for an isolated receptor. (iv) The rate of receptor removal from the cluster is independent of interactions with other receptors in the cluster. Assumptions i–iii are essential assumptions of this model, whereas assumption iv could be altered while preserving the main features of the model. It is important to note that the insertion rate (on rate) and removal rate (off rate) are controlled independently and not governed by a single parameter, which is important for the robust functioning of the model and makes it formally distinct from an Ising spin model of statistical physics (9) (see section 3 of *Supporting Text*, which is published as supporting information on the PNAS web site).

The effect of assumption iv is that the dwell time of a receptor in a cluster is the same as that of an isolated receptor and is independent of cluster interactions. From a biophysical perspective, it might seem more plausible that the on rate is constant and the off rate is neighbor-dependent. Section 2 of *Supporting Text* examines the consequences of this off-rate model and shows that it extends the lifetime of clusters by extending the lifetime of the single

receptors, thus not really addressing the problem. I discuss below how the on-rate model presented here might be justified on a biophysical basis.

For simplicity of implementation, the model is implemented on a square grid. Insertion and removal of synaptic receptors is based on the following specific sequence of events. First, at each time step ( $\Delta t$ ) and for each synaptic site occupied by a receptor, the receptor can be randomly removed from the cluster with a probability  $\mu \Delta t$ . This random removal implies that each receptor has a mean dwell time of  $\tau_{\text{in}} = 1/\mu$  and that its average kinetics are exponential (Fig. 1A); I typically use  $\mu = 1$ , so that times here are measured in units of the mean dwell time of synaptic receptors. Next, at each unoccupied receptor site, a field  $h_1$  is calculated such that its value is the number of occupied neighboring sites minus a lattice repulsion constant  $L_1$ .

$$h_1(i, j) = \left( \sum_{lm} S_{l,m} I(l - i, m - j) - L_1 \right), \quad [4]$$

where  $i, j$  and  $l, m$  are indices of sites in a two-dimensional synaptic surface and  $S_{ij}$  is an occupation variable of the site labeled by indices  $i$  and  $j$  that is 1 if a site is occupied and 0 if not. The function  $I$  is an interaction function. A simple example of an interaction function is the nearest-neighbor function, which assumes that only the four nearest neighbors contribute and has the form

$$I(i, j) = \begin{cases} 1 & \text{if } i^2 + j^2 = 1 \\ 0 & \text{if } i^2 + j^2 \neq 1. \end{cases} \quad [5]$$

Throughout this paper, I assume this simple nearest-neighbor interaction function, which could easily be generalized to more complex local functions.

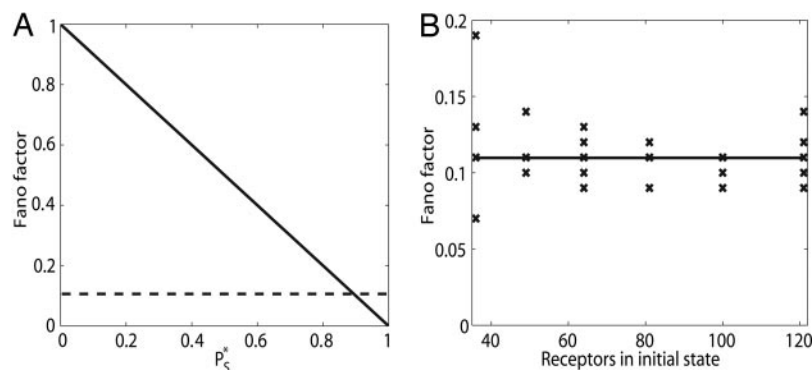
An example of the field calculated for a specific occupation pattern is given in Fig. 1B. The field ( $h_1$ ) calculated determines the probability of inserting a new receptor into an empty site. I implement assumption iii by setting the probability of insertion to be a steeply increasing monotonic function of this field (see Eq. 2). The primary effect of this implementation is that the insertion probability at a site with many neighbors (within a cluster or on its boundary) is orders of magnitude higher than for a site with a small number of neighbors.

Stability characteristics are demonstrated by simulations of cluster dynamics (Fig. 2). Starting from an initial state of 49 receptors (a square of  $7 \times 7$  receptors), single receptors are randomly internalized, and new ones are inserted in their place, resulting in a fluctuating number of receptors at each time step. Examples of the receptor configurations at different time steps are shown in Fig. 2A (see Movie 1, which is published as supporting information on the PNAS web site). The number of receptors in such a cluster fluctuates around stable mean, which is preserved for periods of









**Fig. 6.** The Fano factor for the equilibrium and cluster models. (A) The Fano factor for the equilibrium model plotted from Eq. 4. As the fraction of receptors in the synaptic state ( $P_s^*$ ) increases from 0 to 1, the Fano factor decreases from 1 to 0. (B) Fano factor for the cluster models calculated from the metastable state of simulations with various initial sizes from  $6 \times 6$  to  $11 \times 11$ . The Fano factor shows no significant dependence on size and has a mean of 0.11 (solid line) over all simulations carried out. Five simulations were performed for each initial state, but some of the data points overlap.

The equilibrium model does not address the central issue examined in this paper: how to obtain stable, synapse-specific synaptic efficacies. In the equilibrium model, bidirectional synaptic plasticity can be obtained by changing the kinetic coefficients or the total number of receptors. However, it is not clear how these variables could be changed by a brief and transient plasticity paradigm but maintained for prolonged periods of time. Presumably, this prolonged change would be controlled by molecular processes upstream from the receptors, which are not specified here.

I use the Fano factor, defined as the variance/mean of the receptor number, as a statistical measure of fluctuations. For the equilibrium model,

$$F = (1 - P_s^*), \quad [7]$$

where  $P_s^*$  is the probability of a receptor being in a synaptic state, or, equivalently, the fraction of receptors in the synaptic pool (see section 4 of *Supporting Text*). The dependence of  $F$  on  $P_s^*$  is shown in Fig. 6A.

The Fano factor for the cluster model was calculated for multiple simulations with different initial sizes (Fig. 6B). I carried out five simulations for each initial size and calculated the Fano factor over the initial metastable state. The Fano factor in the cluster model seems independent of the initial size, with an average value over all conditions of 0.11. If the Fano factor were calculated over periods of time spanning transitions from one metastable state to another, it would have a significantly higher value. For the equilibrium model to have Fano factors as small as those of the cluster model, we would have to assume that  $\approx 90\%$  of the receptors reside in the synaptic pool. This prediction is measurable and could be used to distinguish between these two different models. However, the technique used to assess the number of synaptic receptors and their fluctuations must have very small measurement errors, as not to obscure the variability of the receptor number.

## Discussion

The cluster model presented in this paper is proposed as a possible mechanism for long-term stability of synaptic efficacies. I have demonstrated that a synapse formed from a cluster of interacting receptors can have stable efficacies for periods of time that are several orders of magnitude larger than the dwell time of any single receptor in the cluster and that the lifetime of clusters increases rapidly with the number of receptors in the cluster. Estimates of the number of AMPA receptors in a synapse using anatomical methods are on the order of 50–100 (12). Physiological methods estimate that the number of postsynaptic AMPA receptors on spines of CA1 neurons are on the order of 60–190 (13, 14). A cluster with an initial state composed of 81 receptors, well within the plausible range, has a median lifetime of  $>25,000$  time steps. Experiments in which receptor dwell time is indirectly monitored by using overexpression of tagged GluR2 receptors in a slice result in estimated receptor dwell times of 10–30 min (1, 2). Using a receptor dwell time of 20

min, we find that a cluster with 81 receptors in the initial state has a lifetime of  $>1$  year. Although a cluster lifetime of 1 year is still significantly less than the lifetime of memories in a human, the cluster model could be a mechanism to bridge a significant portion of the gap between receptor dwell times and the lifetime of memory. However, because the estimates of receptor dwell time are indirect and because the system used has several properties that could alter the result with respect to a synapse *in vivo*, it might be possible that the real receptor dwell time is much larger. If the receptor dwell time were 1 day, the cluster lifetime would be  $>65$  years.

The model I have presented here is very abstract, and I do not attempt to provide a molecular mechanism that could account for the insertion and removal of receptors. There are many possible mechanistic implementations that could fall under the same family of models. The entity we call a receptor might be a single receptor, but it may also be the complex of a receptor and its associated proteins, or it might include more than one receptor. The removal and insertion of a receptor might be carried out by endocytosis and exocytosis (15) but also by diffusion of receptors within the synaptic membrane (16, 17). The interactions between receptors might be mediated by direct forces between membrane-embedded receptors, the same type of interactions that might lead to aggregation of different proteins (18). However, these interactions are quite likely to depend on the more complex network of postsynaptic proteins associated with the receptors (19, 20), in which case, the properties of these dynamics will be largely independent of the underlying physics of protein aggregation.

In the cluster model proposed here, the receptor's on rate is neighbor-dependent, and the off rate is constant. If the clusters are viewed as aggregates, it might seem that it is more natural to assume the opposite. I have examined the consequences of this off-rate model, and in section 2 of *Supporting Text*, I demonstrate that although it does extend the lifetime of clusters, it does so by extending the dwell time of receptors, not by extending the ratio of cluster lifetime to receptor dwell time. What are possible mechanisms that could support an on-rate model? A constant off rate could result if the molecular machinery that internalizes receptors operates primarily on “tagged” receptors, where the tag might correspond to their phosphorylation state (21). If this tagging procedure proceeds at a fixed rate, then the off rate will be constant. A neighbor-dependent on rate could come about if the nearby receptors act to somehow reduce the energetic cost of inserting a receptor in their vicinity, which could occur directly by electromagnetic shielding or indirectly through the network of synaptic proteins linked to the receptors. Another possibility is that the on rate is an effective rate brought about by the formation of a diffusive trap caused by clusters. This alternative is consistent with recent experimental findings that show that receptors diffuse at two distinct rates, with a much lower rate spatially coincident with locations of synaptic contacts (16, 17). The consequences of this diffusive trap

